"A STUDY OF FUZZY LATTICE CONCEPTS OVER ALGEBRAIC STRUCTURES WITH OPERATORS"

A
Synopsis
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Title of proposed work plan

"***A Study of Fuzzy lattice Concepts over Algebraic Structures with Operators***"

Abstract of proposed work plan /problem.

Algebra particularly lattice is a branch of mathematics concerning the study of the rules of operations and relations and the construction and concepts arising from them including algebraic structures. Almost all the branches of pure mathematics lead to an algebraic structure. Algebraic structure provides a useful framework for the study of the subject. In particular lattice can be treated as an algebraic structure with operations '∧' & '∨' satisfying some rules. A number of researchers have made research for lattice as an algebraic structure. But when lattice is considered as a fuzzy set instead of a crisp set there is a scope for research. So many researchers have given their contribution to this field. Also there is a concept group with operators. Therefore the topic "*A Study of Fuzzy Lattice Concepts over Algebraic Structures with operators*"is selected for the research. In this we proposed to study the operator effect on various fuzzy lattice algebraic structures like group, normal subgroup, ring, ideal etc.

1.Introduction to proposed research topic.

The study is proposed to introduce thenotionoffuzzy lattice ordered algebraic structure with operators and to investigate some of its basic properties.The proposed study is about to introduce action of more than one operators on an algebraic structure and its generalization for n operators. These operator actions will be observed on groups, normal subgroups, rings, ideals etc. The effect will be seen on homomorphism, its image and pre-image of groups. Recently the researchers introduced the notion of fuzzy lattice ordered m-groups with a single operator. In this, the study is proposed for two and more than two operators and generalized operator theory actions on fuzzy ordered lattices.

2. A review of the literature and development in the subject

A fuzzy algebra has become an important branch of research. Rosenfeld[7]1971used the concept of fuzzy set theory due to Zadeh[11]1965.Sincethen the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. Ajmal and Thomas[1] initiated such types of study in the year1994.Itwaslatter independently established by Ajmal that the set of all fuzzy normal subgroups of a group constitute a sublattice of the lattice of all fuzzy subgroups of a given group and is Modular. Nanda[6] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy subgroups and fuzzy normal sub groups are explored. SatyaSaibaba[8] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l-groups. Goguen[2] replaced the valuation set[0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets.Solairaju and Nagarajan[9] introduced the concept to fuzzy lattice valued Q-fuzzy sub-modules

over near rings with respect to T-norms. Marudai &Rajendran[5]modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [3] introduced concept of fuzzy groups with operator. Then Subramanian, Nagrajan & Chellappa [10] extended the concept to m fuzzy groups with operator. Lokhande and Makandar [4] introduced the notion of fuzzy lattice ordered m-groups and investigated some of its basic properties.They also studied the homomorphic image,pre-image of fuzzy lattice ordered m-groups, arbitrary family of fuzzy lattice ordered m-groups and introduced the notion of sensible fuzzy lattice ordered m-groups in groups using T-norms and some related properties of lattices.

Fuzzy group - Let $\mu: X \rightarrow [0, 1]$ be a fuzzy set & $G \in \beta(X)$ = Set of all fuzzy sets on X. A fuzzy set μ on G is called a fuzzy group if i) $\mu(x\,y) \geq \min\{\mu(x), \mu(y)\}$ ii) $\mu(x^{-1}) \geq \mu(x)$, for all $x, y \in G$.

Lattice ordered group - A lattice ordered group is a system

$(G, \leq)$ if i) $(G, \cdot)$ is a group ii) $(G, \leq)$ is a lattice. iii) $x \leq y$ implies $a\ x\ b \leq a\ y\ b$ (Compatibility) For $a, b, x, y \in G$

Fuzzy lattice ordered group- Let $\mu: X \rightarrow [0, 1]$ be a fuzzy set & G is a lattice ordered group, $G \in \beta(X)$. A function μ on G is said to be a fuzzy lattice ordered group if

i) $\mu(x\,y) \geq \min\{\mu(x), \mu(y)\}$

ii) $\mu(x^{-1}) \geq \mu(x)$

iii) $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$

iv) $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in G$

m group-Let G be a group, M be any set if $m\,x \in G$. for all $x \in G$, $m \in M$. Then G is called a m group.

Fuzzy m group- Let μ: X$\rightarrow$ [0, 1] be a fuzzy set & G be a M group . A fuzzy set on G, G ϵ þ(X)) is called a fuzzy m group if i) μ (m(x y)) $\geq$ min {μ (m x), μ (my)} ii) μ (mx^{-1}) $\geq$ μ (m x)for all x, yϵ G, m ϵ M

Lattice ordered m group - A lattice ordered m group is a system (G, $\leq$) if i) (G, $\cdot$) is a m-group ii) (G,$\leq$) is a lattice . iii) x $\leq$ y implies a x b $\leq$ a y b (Compatibility) For a, b, x, yϵ G

F**uzzy lattice ordered m-group**- μ: X to [0, 1], G ϵ þ(X) be a lattice ordered m group, M C X. A function μ on G is said to be a fuzzy lattice ordered m-group if

i) μ (m(x y)) $\geq$ min {μ (m x), μ(my)}

ii) μ ((m x) $^{-1}$) $\geq$ μ (m x)

iii) μ (m x v my) $\geq$ min {μ (m x), μ(m y)}

iv) μ (m x $\wedge$ my) $\geq$ min {μ (m x), μ(my)} For all x, y ϵ G

Proposed structures

m n group-Let G be a group, M ,N be any set if m x ϵ G , n xϵ G. for all x ϵ G, m ϵ M. n ϵ N Then G is called a m n group.

Fuzzy m n group- Let μ: X$\rightarrow$ [0, 1] be a fuzzy set & G be a m ngroup . A fuzzy set on G, G ϵ þ(X)) is called a fuzzy m n group if i) μ (mx ny)) $\geq$ min {μ (m x), μ (ny)} ii) μ (($mxny)^{-1}$) $\geq$ μ (m x n y)for all x, yϵ G, m ϵ M ,m ϵ N

Lattice ordered m n group - A lattice ordered m n group is a system (G, $\leq$) if i) (G, $\cdot$) is a m n -group ii) (G,$\leq$) is a lattice . iii) x $\leq$ y implies a x b $\leq$ a y b (Compatibility) For a, b, x, yϵ G

Fuzzy lattice ordered m-n group- μ: X to [0, 1], G ϵ þ(X) be a lattice ordered m-n group, M ,N C X. A function μ on G is said to be a fuzzy lattice ordered m-n group if

i) μ (mx.ny) $\geq$ min { μ (mx), μ (ny)}

ii) μ {($mx.ny)^{-1}$}$\geq$ μ {mx ny}

iii) $\mu (mx \vee ny) \geq \min \{ \mu (mx), \mu (n y)\}$

iv) $\mu (mx \wedge ny) \geq \min \{ \mu (mx), \mu (n y)\}$ For all $x \in G, m \in M, n \in N$

3. Objectives of the research / proposed hypothesis.

1) To study the fuzzy lattice concepts defined over different algebraic structures with operators particularly group, normal subgroup, rings, ideals etc.

2) To generalize the concepts for algebraic structure with n operators.

3) To study the various properties of algebraic structures on the new defined structure

4) To study the homomorphism and its properties with respect to the new defined structure.

4. Methodology to be used and adopted.

a)Statement of Hypothesis: The concept of fuzzy lattice ordered group can be extended to fuzzy lattice ordered m group,fuzzy lattice ordered m ring, fuzzy lattice ordered m n group,fuzzy lattice ordered m n ring and fuzzy lattice ordered n operator group-ring.

b)Method of data Collection: Secondary data will be collected from books, published research papers, referred journals and information collected from internet.

5. Importance of the study / Society Application

Lattice structure has been found to be extremely important in the areas of quantum logic, Erogodic theory, Reynolds's operations, Soft Computing, Communication system, Information analysis system, artificial intelligences and physical sciences.

Fuzzy logic has been used in numerous applications such as facial pattern recognition, air conditioners, washing machines, vacuum cleaners, antiskid braking systems, and transmission systems, control of subway systems and unmanned helicopters, knowledge-based systems for multiobjective optimization of power systems.

6.Proposed work plan with a tentative chapter scheme

Chapter 1. Introduction to the fuzzy sets, fuzzy group, lattices, fuzzy lattices, lattice ordered sets, lattice ordered groups, fuzzy lattice ordered groups, m groups, m n groups, fuzzy lattice ordered m groups, fuzzy lattice ordered mn groups.

Chapter 2.Examples of fuzzy lattice ordered m group and m n groups

Chapter 3. Properties of fuzzy lattice ordered m group and m n groups

Chapter 4.Applying the concepts to various algebraic structures like normal groups, ideals, rings

Chapter 5. Generalization of the concepts to n operators.

Chapter 6. Findings and conclusion

7.Year wise plan of work and targets to be achieved.

Year 1-Completion of course work and course work exam. Preparation of synopsis and finalization through concerned authority.

Year 2-Defining a new algebraic structure and study the effect on various algebraic structures. Development of examples of m algebraic structure, m n algebraic structure and testing the validity of the new structure.

Year 3- Study of new structure over normal subgroups, ideals, ring etc

Year 4- Study of Generalized concept, report writing, embossing and submission.

8. Bibliography / references

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